Eur. Phys. J. A **24**, 85–92 (2005)
DOI 10.1140/epja/i2004-10128-3

THE EUROPEAN PHYSICAL JOURNAL A

Microcanonical studies on isoscaling

Ad.R. Raduta[a]

National Institute of Nuclear Physics and Engineering, Bucharest, P.O. Box MG6, Romania

Received: 23 July 2004 / Revised version: 29 November 2004 /
Published online: 8 February 2005 –
Communicated by G. Orlandini

Abstract. The exponential scaling of isotopic yields is investigated for sources of different sizes over a broad range of excitation energies and freeze-out volumes, in both primary and asymptotic stages of the decay in the framework of a microcanonical multifragmentation model. It was found that the scaling parameters have a strong dependence on the considered pair of equilibrated sources and excitation energy and are affected by the secondary particle emission of the break-up fragments. No significant influence of the freeze-out volume on the considered isotopic ratios has been observed. Deviations of microcanonical results from grandcanonical expectations are discussed.

PACS. 25.70.Pq Multifragment emission and correlations – 24.10.Pa Thermal and statistical models

1 Introduction

Many nuclear physics experiments realized in the last years revealed an interesting scaling property of the isotopic yield ratios obtained from the disintegration of equilibrated sources of similar sizes, with close values of excitation energy per nucleon and temperature but different isospin values as a function of the isotopic composition of the emitted cluster [1,2]. This property has been called isoscaling and can be expressed mathematically by the formula

$$R_{21}(Z,N) = Y_2(Z,N)/Y_1(Z,N) = C\exp(\alpha N+\beta Z), \quad (1)$$

where $Y_i(Z,N)$ denotes the yield of the isotope (Z,N) obtained from the decay of the excited nuclear system "i" and C, α and β are scaling parameters.

If one classifies the nuclear reactions which manifest isoscaling function of the projectile energy, one can say that isoscaling has been observed in reactions induced by projectiles whose energies range from a few MeV/nucleon to several GeV/nucleon. In terms of reaction mechanisms, isoscaling has been obtained in evaporation reactions (*e.g.*, ^{4}He + ^{116}Sn, ^{124}Sn at 50 MeV/nucleon bombarding energy [3]), deep inelastic reactions (*e.g.*, ^{16}O + ^{232}Th, ^{16}O + ^{197}Au at 8.6 MeV/nucleon [4], ^{86}Kr + ^{112,124}Sn and ^{86}Kr + ^{58,64}Ni at 25 MeV/nucleon [5]) and, recently, in fission reactions (*e.g.*, n + ^{233,238}U at 14 MeV/nucleon [6]). The biggest amount of experimental isoscaling data comes from multifragmentation reactions [1,2,7–10] and the great interest in these studies was motivated by the possibility to infer information on the nuclear equation of state, and in particular on the asymmetry term, from the isoscaling parameters [8,11].

It is interesting to specify at this point that, relatively easy to understand for nuclear reactions involving equilibrated compound nuclear systems [7], isoscaling is compatible also with dynamical fragment formation as was recently demonstrated in refs. [12,13].

The aim of the present paper is to investigate isoscaling in multifragmentation reactions and, more precisely, the results produced within a microcanonical framework. In order to learn how sensitive the isoscaling parameters are with respect to the observables characterizing the source's state, we considered sources whose nucleon number varies from 40 to 200, with excitation energies between 2 and 15 MeV/nucleon, freeze-out volumes ranging from $3V_0$ to $10V_0$ in both primary and asymptotic stages of the decay. The relevance of such study consists in the fact that the most suitable statistical ensemble to describe the decay of an isolated excited nuclear system counting at most several hundreds of nucleons is the microcanonical ensemble. The importance of using models which can mimic as well as possible the real physical case is demonstrated by the fact that the best description of the experimental data was obtained by the predictions of microcanonical multifragmentation models [14–16].

Section 2 makes a review of the grandcanonical argumentation of isoscaling in nuclear multifragmentation. Section 3 presents microcanonical results on isoscaling obtained for different mass systems with excitation energies ranging from 2 to 15 MeV/nucleon at different freeze-out volumes in both primary and asymptotic stages of the

[a] e-mail: araduta@ifin.nipne.ro

decay. The sensitivity of the isoscaling parameters to observables characterizing the state of the source is discussed in detail. Deviations of microcanonical results from grand-canonical expectations are commented. The conclusions are drawn in sect. 4.

2 Isoscaling in nuclear multifragmentation

Isoscaling in multifragmentation was for the first time evidenced for the light emitted fragments ($A \leq 18$) obtained in the reactions ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{124}Sn at 50 MeV/nucleon bombarding energy [1] and confirmed by many other reactions induced by relativistic light particles (*e.g.*, p, d, α + ^{112}Sn and ^{124}Sn at incident energies between 660 MeV and 15.3 GeV [8]) or heavy ions (^{124}Sn + ^{64}Ni and ^{112}Sn + ^{58}Ni at 35 MeV/nucleon [9]).

The theoretical interpretation of isoscaling in the multifragmentation energy domain was given using a grand-canonical ensemble [7]. It was demonstrated that in the grandcanonical limit the yield of a primary fragment (Z,N) obtained from the decay of a thermally equilibrated system characterized by the temperature T can be written as [17]

$$Y(Z,N) \propto \exp(B(Z,N)/T)\exp(N\mu_n/T + Z\mu_p/T), \quad (2)$$

where $B(Z,N)$ is the binding energy of the emitted fragment (Z,N) and $\mu_{\rm n}$ and $\mu_{\rm p}$ are the neutron and proton chemical potentials. Taking into account that the two equilibrated systems have the same temperature, excitation energy per nucleon and similar sizes the proportionality factors from eq. (2) have been considered practically identical in the two cases. Thus, one obtains for the ratio of the yields of any isotope resulting from the decay of the two sources the expression of eq. (1), where

$$\alpha = \Delta\mu_{\rm n}/T, \quad (3)$$

and

$$\beta = \Delta\mu_{\rm p}/T, \quad (4)$$

where $\Delta\mu_{\rm n(p)} = \mu_{\rm 2n(p)} - \mu_{\rm 1n(p)}$.

Despite the fact that the suitability of a grandcanonical approximation is hard to be justified for systems having at most few hundreds particles, the success of the above formulas was due to their simplicity and to the fact that no contradiction came from experimental data or various statistical multifragmentation models (*e.g.*, microcanonical [14] and canonical [18] versions of SMM, Expanding Emitting Source [19]) currently used in data analyses [11].

Concerning the slope parameters α and β, several values have been reported in the literature. This fact is expected because it is known that particle formation probabilities have explicit dependence on all observables which characterize an excited source (mass A, charge Z, excitation energy $E_{\rm ex}$, freeze-out volume V). It is also known that the sequential particle emission from the excited primary fragments modifies the isotopic yields. This evaporation process may also affect the values of α and β.

3 Isoscaling from a microcanonical perspective

Despite the fact that, contrary to the grandcanonical case, no microcanonical model provides analytical expressions for cluster yields, the occurrence of isoscaling in a microcanonical model can be understood starting from the shape of the isotopic yields corresponding to a given element [8]. Thus, it has been observed that charge distributions of fragments with fixed values of the neutron number ($Y(Z,N)|_N$) and N distributions of fragments with fixed values of the proton number ($Y(Z,N)|_Z$) can be approximated by Gaussian functions. The occurrence of such Gaussian-like distributions within a microcanonical model can be regarded as a natural consequence of complete equilibration. Using the generic notation $Y(x,y)|_y$ for the distribution of the observable x when y is kept fixed (where $x = N, Z$ and $y = Z, N$), one can write

$$Y(x,y)|_y = C\exp(-(x - x_{\rm med}(y))^2/2\sigma(y)^2). \quad (5)$$

The mean value $x_{\rm med}(y)$ gives the isotope produced with the largest probability, while the variance $\sigma(y)$ is a measure of how uniform is the population of fragments having different isospin values. The statistical meaning of these quantities involves an obvious dependence on observables determining the ensemble's state. Since we deal with a microcanonical approximation, these observables are the mass, volume and excitation energy of the multifragmenting source.

Using eq. (5), the ratio of isotopic yields writes

$$\begin{aligned} R_{21}(x, y = {\rm const}) &= \frac{Y_2(x,y)|_y}{Y_1(x,y)|_y} \\ &= \exp\left[-\frac{x^2}{2}\left(\frac{1}{\sigma_2(y)^2 - \sigma_1(y)^2}\right)\right. \\ &\quad + x\left(\frac{x_{\rm med2}(y)}{\sigma_2(y)^2} - \frac{x_{\rm med1}(y)}{\sigma_1(y)^2}\right) \\ &\quad \left. - \left(\frac{x_{\rm med2}(y)^2}{2\sigma_2(y)^2} - \frac{x_{\rm med1}(y)^2}{2\sigma_1(y)^2}\right)\right]. \end{aligned} \quad (6)$$

Taking into account that the two considered sources have similar sizes, temperatures and freeze-out volumes, the isotopic distributions corresponding to any cluster are expected to have equal variances ($\sigma_1(y) = \sigma_2(y) = \sigma(y)$). Thus, in eq. (6) the quadratic term in x vanishes leading to a linear dependence of the yield ratio on the observable x of the emitted cluster.

From eq. (6) one can express the α and β slope parameters in terms of mean values and variances of isotopic distributions,

$$\alpha(Z) = \frac{1}{\sigma(Z)^2}\left(N_{\rm med2}(Z) - N_{\rm med1}(Z)\right), \quad (7)$$

$$\beta(N) = \frac{1}{\sigma(N)^2}\left(Z_{\rm med2}(N) - Z_{\rm med1}(N)\right). \quad (8)$$

From eqs. (7) and (8) it results that α and β may depend on the size of the emitted cluster. A necessary condition to obtain slope parameters independent of the size of the emitted cluster as in eq. (1), is to have a constant value for the ratio of the shift between the isotopic distributions produced by the two sources and the square of the distributions' variance. As we shall see in the following sections, this holds with good approximation for the light clusters emitted by relatively large equilibrated sources but does not hold for heavy fragments emitted by heavy sources and clusters emitted by light multifragmenting sources.

In the following we shall investigate isoscaling over a large range of masses, excitation energies and freeze-out volumes of the multifragmenting sources in order to determine the dependence of the slope parameters α and β on the source parameters, the influence of the secondary decays on α and β and, in the case of small sources, the dependence of α and β on the size of the emitted cluster. To serve this goal, we use the microcanonical multifragmentation model presented in detail in refs. [20,16].

3.1 Model overview

The microcanonical multifragmentation model [20,16] has two distinct stages: the break-up stage and the sequential particle evaporation stage.

The most important part of the model is the so-called break-up stage which aims to describe the explosion of the equilibrated nuclear source. In order to mimic as well as possible the physical situation of a small isolated system with fixed excitation energy, the model tries to obey sharply to microcanonical constrains. Thus, the fixed observables characterizing the state of the equilibrated source are mass, charge, total energy E, total momentum $\mathbf{P}$ (= 0 in the center of mass (c.m.) frame) and total angular momentum $\mathbf{L}$ (= 0 for non-rotating systems). The freeze-out volume V is determined by the spherical container in which fragments are generated.

The standard version of the model allows treatment of freeze-out volume according to two different scenarios [16]: hard spherical non-overlapping fragments placed in a spherical container and spherical container with free volume parameterization [21]. For a faster simulation we considered the second case. The mathematical expression of the free volume is

$$V_{\mathrm{free}} = \prod_{i=1}^{N_{\mathrm{fr}}} \left(V - i\frac{V_0}{N_{\mathrm{fr}}} \right), \tag{9}$$

where N_{fr} is the number of fragments in a given event and V_0 is the volume of the nuclear system at normal density.

The price to pay for a model aiming to respect all microcanonical constrains is the impossibility to obtain formulas analytically tractable. The key quantity of the model is the statistical weight of a configuration C,

$$W_C \propto \frac{1}{N_C!} \prod_{n=1}^{N_C} \left(\Omega \frac{\rho_n(\epsilon_n)}{h^3} (mA_n)^{3/2} \right) \times \frac{2\pi}{\Gamma(3/2(N_C-2))} \frac{1}{\sqrt{(\det I)}} \frac{(2\pi K)^{3/2 N_C - 4}}{(mA)^{3/2}}, \tag{10}$$

where N_C is the number of fragments corresponding to configuration C, Ω is the accesible volume, I is inertial tensor of the system and K is the available kinetic energy. The index n denotes the fragments in each configuration.

The average value of any observable Q is calculated via a Metropolis type simulation according to the expression,

$$\langle Q \rangle = \frac{\sum_C Q_C W_C}{\sum_C W_C}. \tag{11}$$

Because the observables studied in the present paper are functions of isotopic yields they are strongly dependent on all specific details of the model (binding energy parameterization, allowed isospin asymmetry of primary fragments, employed level density formula, etc.). The break-up fragments allowed to be formed are all isotopes included in the mass table of ref. [22].

The fragments' binding energy was calculated according to the liquid-drop parameterization:

$$\begin{aligned} B(A,Z) = {} & 15.4941(1-1.7826I^2)A \\ & -17.9439(1-1.7826I^2)A^{2/3} \\ & -0.7053Z^2A^{-1/3} + 1.1530Z^2/A, \end{aligned} \tag{12}$$

where $I = (A-2Z)/A$.

Fragments with mass smaller than 4 are considered without internal excitation energy; larger fragments are allowed to carry an excitation energy (ϵ) upper limited by the binding energy according to the following level density formula:

$$\rho(\epsilon) = \frac{\sqrt{\pi}}{12a^{1/4}\epsilon^{5/4}} \exp(2\sqrt{a\epsilon}) \exp(-\epsilon/\tau), \tag{13}$$

with $a = 0.114A + 0.098A^{2/3}$ MeV^{-1} [23] and $\tau = 9$ MeV. The factor $\exp(-\epsilon/\tau)$ takes into account the decrease of the excited level lifetime with the increasing excitation energy.

The excited primary fragments obtained in the break-up stage are allowed to decay by sequential particle emission (second stage of the model) as in ref. [16]. The range of the evaporated particles is considered up to $A = 16$. As already stated in the literature [24,25] by modifying isotopic yields, secondary particle production mechanisms may alter isoscaling parameters.

Despite the fact that the employed parameterizations are expected to affect the specific values of α and β, they do not affect the general behavior of isoscaling parameters as a function of observables characterizing the sources' state such as the conclusions of the study are general and valid.

3.2 General microcanonical results

In order to verify to what extent isoscaling holds within a microcanonical framework and, if this is the case, to make a systematic study of all source parameters which might have an influence on the isoscaling parameters, we considered pairs of equilibrated sources with masses ranging

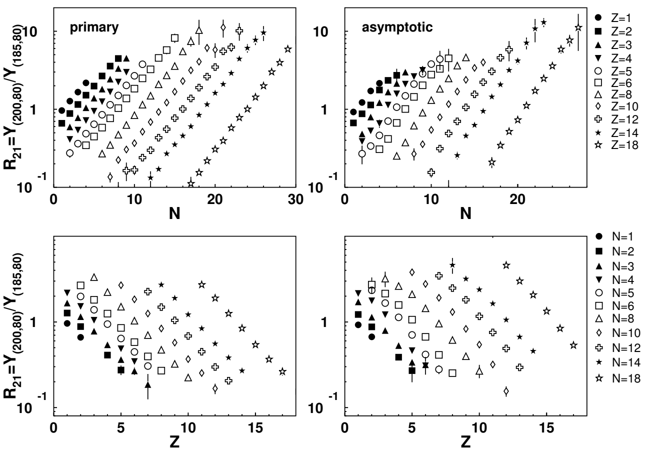

Fig. 1. Isotopic yield ratios are plotted in logarithmic scale as a function of N (upper panels) and Z (lower panels) for two equilibrated sources (200, 80) and (185, 80) with the excitation energy $E_{\mathrm{ex}} = 5$ MeV/nucleon and the freeze-out volume $V = 6V_0$ in both primary (left panels) and asymptotic stages of the decay (right panels). The considered sequence of $N(Z)$ is $1, 2, 3, 4, 5, 6, 8, 10, 12, 14, 18$.

from $A = 40$ to $A = 200$, excitation energies ranging from 2 to 15 MeV/nucleon, freeze-out volumes ranging from $3V_0$ to $10V_0$ in both primary and asymptotic stages of the reaction.

For all considered cases we obtained isoscaling in the sense that $\log R_{21}(Z, N)$ shows an almost perfect linear dependence as a function of N and Z.

As general observations, one may say that the quality of the isotopic scaling is better in the break-up stage than in the asymptotic stage of the decay and the maximum size of the emitted cluster for which isoscaling still holds decreases with increasing excitation energy. We think that the more modest quality of the isoscaling in the asymptotic stage is an artifact of the simplified manner in which the secondary decays have been implemented. We adopt the widely used convention to denote with the index "2" the more neutron-rich system and with the index "1" the more neutron-poor system. In this situation the value of α is always positive because more neutron-rich clusters will be produced by the neutron-richer source and the value of β is always negative. To avoid possible effects of different magnitudes of Coulomb interaction on isotopic distributions, the considered pairs of sources have the same proton number and different mass numbers.

To illustrate how isoscaling looks like in the case of this microcanonical model, in fig. 1 we present the isotopic yield ratios as a function of N and Z for the pair of equilibrated sources (200, 80) and (185, 80) at 5 MeV/nucleon excitation energy and a freeze-out volume 6 times larger than the nuclear volume at normal density. As results from the figure, isoscaling holds in the strict sense of eq. (1), namely linear behavior of $\log R_{21}$ as a function of N and Z and constant value of C. Special attention should be drawn to the fact that in fig. 1 distances from the points corresponding to different values of $Z(N)$ are not equal because the considered sequence of $Z(N)$ does not contain exclusively consecutive numbers (see legend).

It is interesting to notice that a linear behavior of $\log R_{21}$ is obtained also for emitted clusters much heavier than those for which the experimental analysis has been possible. As a technical detail, we stress that although for a given element the linear behavior of $\log R_{21}(N)$ holds for almost all obtained isotopes, for the fitting procedure only the most stable fragments have been selected.

3.3 Dependence of α and β on excitation energy

Since, as is known from the early days of multifragmentation, the excitation energy of the nuclear source induces strong modifications on all fragment multiplicities, it is expected to affect also isotopic yield ratios.

In fig. 2 α and β for the equilibrated sources (200, 80) and (185, 80) are represented, with $V = 6V_0$ as a function of the excitation energy. It must be specified that, if not mentioned in a different manner, along this paper α and β are calculated such as to obtain the best fit for fragments with $2 < Z < 9$ and, respectively, $2 < N < 9$. This choice is made in order to realize an analysis as much similar as possible to the way in which the slope parameters have been obtained from experimental data and to have the same amount of considered data for all excitation energies and all considered source sizes.

One can see that both parameters have a monotonic dependence on the excitation energy, their absolute value decreasing with the temperature. Thus, increasing the excitation energy from 2 MeV/nucleon to 15 MeV/nucleon α diminishes by a factor of 2.5 and $|\beta|$ diminishes by a factor of 3.5 This effect can be understood having in mind that an increase in excitation energy will result in a more uniform population of the isotopes corresponding to a given element thus washing out the effect of the isotopic composition of the source. In terms of eqs. (7) and (8), an increasing excitation energy will lead to a shift of the $Y(x, y)|_y$ distributions toward smaller values of x_{med} such as $(x_{\mathrm{med2}} - x_{\mathrm{med1}})$ slightly diminishes together with an enhancement of the distributions' variance σ. Since both numerator and denominator from eqs. (7) and (8) act in the same sense, the decrease of α and $|\beta|$ with the excitation energy of the sources is obvious.

These results are in qualitative agreement with those of refs. [8, 10].

3.4 Effect of secondary decays on α and β slope parameters

An important problem that manifests whenever we need to infer the break-up information from the available experimental multifragmentation data is the reconstruction of primary observables affected by the secondary particle emission. This problem may be even more important when one operates with very sensitive quantities like the slope parameters α and β. It was stated in the literature

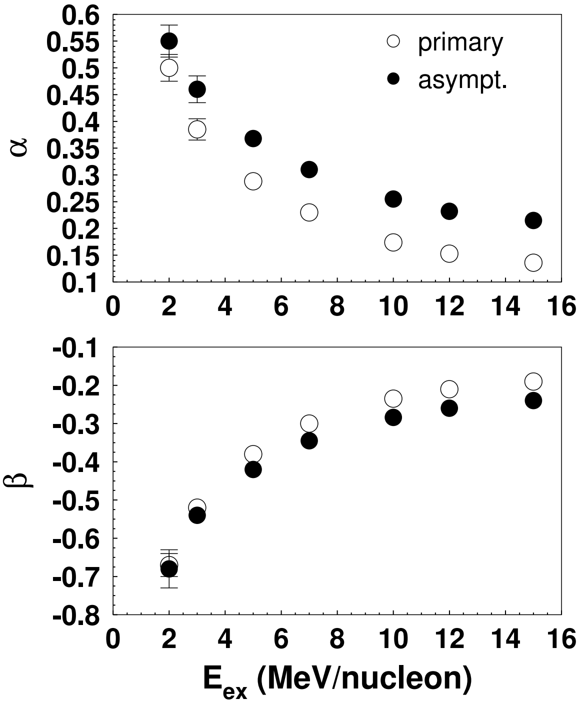

Fig. 2. Slope parameters α and β as a function of excitation energy for primary (open symbols) and asymptotic (full symbols) stages of the decay. The considered equilibrated sources are (200, 80) and (185, 80). The freeze-out volume is $V = 6V_0$.

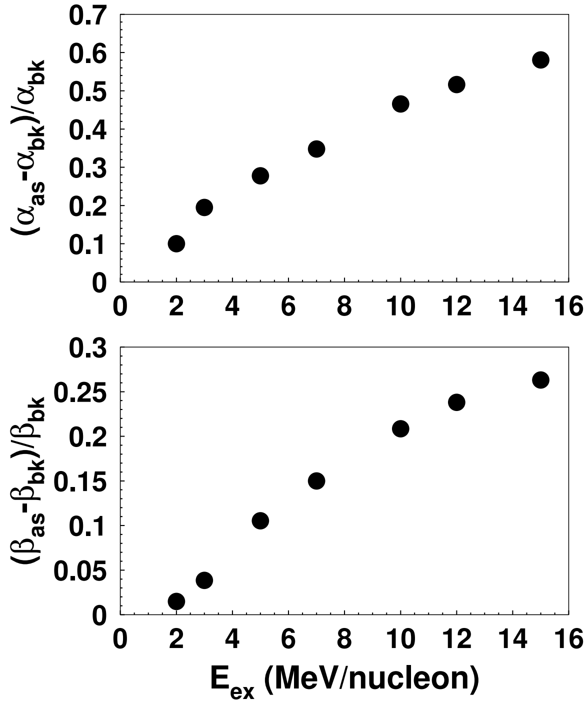

Fig. 3. Effect of secondary decays on the slope parameters α and β as a function of excitation energy. The considered equilibrated sources are (200, 80) and (185, 80) with $V = 3V_0$.

that when the two considered sources have almost identical parameters excepting the isospin, the effects of sequential evaporation cancel one another leading to small modifications of the break-up stage results [2]. On the other hand, estimations performed with accurate models like SMM [14] indicate that modifications of the slope parameters due to evaporation may manifest with different magnitudes depending on how sequential evaporation is implemented [8, 11].

To check the evaporation effect on isoscaling within the present model in fig. 2 α and β are plotted in the break-up and asymptotic stages of the decay for the equilibrated sources (200, 80) and (185, 80), with $V = 6V_0$ as a function of the excitation energy. As displayed by the figure, over the entire energy range the absolute values of α and β are larger in the asymptotic stage of the decay in comparison to the break-up stage.

To get an insight on what happens during the sequential particle emission from primary excited fragments it is necessary to focus on the evaporation process. Given the fact that for all excitation energies neutrons are emitted with the most important probability, the final cold fragments are more symmetric than their break-up ancestors. This means that the isotopic distribution of a given Z shifts toward smaller values of N_{med}. Since the amplitude of this shift is larger for the neutron-richer source, it means that the value $(N_{\text{med2}}(Z) - N_{\text{med1}}(Z))$ in the asymptotic stage is smaller than the one corresponding to the break-up stage. This apparent trend of isoscaling parameters to decrease after particle evaporation is annihilated by a strong narrowing of isotopic distributions (the more asymmetric nuclei have small survival probabilities after secondary decays) so that in the end the values of α and $|\beta|$ increase with respect to their break-up values. This result is counterintuitive because at a first glance one could expect secondary decays to wash out the isotopic differences of the sources.

From fig. 2 it is obvious that secondary decays affect more α than β. This result can be understood taking into account that the decrease of the width of $Y(Z,N)|_Z$ distributions caused by neutron evaporation is more important than the narrowing of $Y(Z,N)|_N$ distributions caused by less important charged particle emission.

One can see from fig. 2 that the difference between the asymptotic and break-up values increases up to 5 MeV/nucleon excitation energy and then stays roughly constant, but taking into account that the increase in excitation energy results in smaller slope parameters, this means that the relative influence of the secondary decays on α and β grows with energy. To illustrate the relative modifications brought by the secondary decays to the slope parameters in fig. 3 the deviation between asymptotic and primary values relative to the break-up results *versus* excitation energy is plotted. The increasing rate is roughly constant for whole considered domain.

3.5 Slope dependence on the freeze-out volume

To identify a possible freeze-out volume effect on the slope parameters, isoscaling was analyzed for the previously mentioned pair of sources ((200, 80) and (185, 80)) at excitation energies increasing from 2 to 15 MeV/nucleon at $V = 3V_0$, $V = 6V_0$, $V = 10V_0$ in both primary and asymptotic stages of the decay. Despite the fact that all isotopic distributions present a strong dependence on the volume of the excited source, for the considered cases no significant freeze-out volume dependence of α and β was identified.

The impossibility to get an analytic expression of isotopic yields within a microcanonical model makes very hard to estimate the effect of freeze-out volume (or any other observable characterizing the state of the source) on isotopic ratios. However, if true, the independence of the slope parameters from the freeze-out volume is of particular importance for experiments aiming to extract the symmetry term of the binding energy from isotopic yields. Indeed, no matter how the method for selecting collisions is, it is hard to believe that the ensemble of equilibrated nuclear systems formed in heavy-ion reactions correspond to one value of the freeze-out volume, but rather to a distribution. Since there is no method to select fragmentation events according to their volume, volume-dependent α and β would make impossible the extraction of the asymmetry term.

Reference [11] makes use of a canonical version of SMM [18] and reaches the conclusion that for all considered temperatures ($T = 4, 5, 6$ MeV) α increases monotonically with the freeze-out density: by increasing ρ/ρ_0 from 0.1 to 1 α increases by a factor of two. The opposite conclusions of ref. [11] and the present study are produced by the in principle different basic hypothesis of each model. Taking into account the implications of the volume dependence of isoscaling parameters, a definite answer to this problem would be of much interest.

3.6 Slope dependence on the considered sources

All the above discussion suggests a strong dependence of the fragment production on the parameters characterizing the equilibrated state of the sources. A natural question to arise at this point is how sensitive are the slope parameters on the considered sources once one keeps fixed the atomic number, the freeze-out volume and the excitation energy.

To answer this question, α and β have been calculated for different pairs of sources (chosen from the set (180,80), (185,80), (190,80) and (200,80)) at $E_{\rm ex} = 5$ MeV/nucleon and $V = 3V_0$. The result we obtained is that the slope parameters are not dictated by the difference of isospin asymmetry between the two sources, but by the difference between the binding energy of the sources. To illustrate this result, we plot in fig. 4 the monotonic dependence of α and β on the difference between the binding energies of the equilibrated sources.

This result is in qualitative agreement with some experimental data. Reference [13] reports a modification

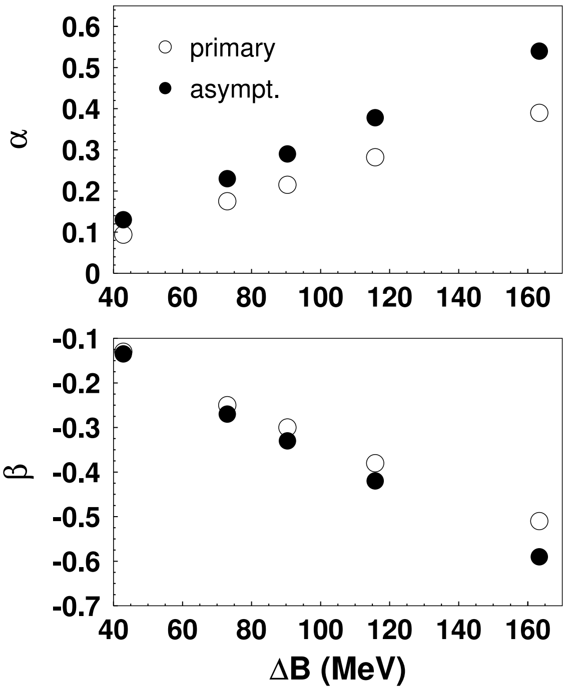

Fig. 4. Slope parameters α and β as a function of the difference between the binding energies of the equilibrated sources. Open symbols correspond to break-up isotopic yields and full symbols correspond to isotopic yields after sequential particle emission. The considered equilibrated sources are characterized by $Z = 80$, $E_{\rm ex} = 5$ MeV/nucleon and $V = 3V_0$.

of the absolute values of α and β by a factor of two when calculated using (^{112}Sn + ^{124}Sn/^{112}Sn + ^{112}Sn) nuclear reactions instead of (^{124}Sn + ^{124}Sn/^{112}Sn + ^{112}Sn) at 50 MeV/nucleon. Similar results were recently obtained by studying isoscaling in (^{58}Fe + ^{58}Ni/^{58}Ni + ^{58}Ni) and (^{58}Fe + ^{58}Fe/^{58}Ni + ^{58}Ni) reactions at 30, 40 and 47 MeV/nucleon [10].

Contrary to the cases of a grandcanonical model or a sequential particle emission model, where one can easily express isotopic multiplicities as a function of the binding energy of the parent nucleus, within a microcanonical model the effect of binding energy can be estimated only in an intuitive way. Thus, the more similar are the two considered equilibrated sources from the point of view of their binding energies, the closer are the values of the total available energy of each system for a given value of the excitation energy. This will lead to similar populations of the configuration space of each system. In this case, all observables (including isotopic yields) are expected to have close values. Close values of isotopic yields will obviously produce small values of α and β.

3.7 Slope dependence on the source size

The only parameter characterizing the equilibrated source which was kept fixed thorough the above discussion is the

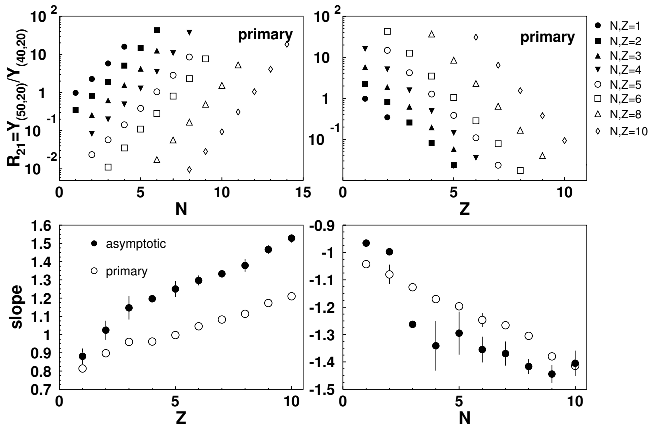

Fig. 5. Upper panels: isotopic yield ratios for the primary decay of (50, 20) and (40, 20) at $E_{\mathrm{ex}} = 5$ MeV/nucleon and $V = 6V_0$ as a function of N (left panel) and Z (right panel). Lower panels: dependence of the slope parameters α and β of the size of the considered emitted cluster. Here α (β) represents the slope of $\log R_{21}(Z,N)$ *versus* N (Z) for each considered value of Z (N). The considered sequence of $N(Z)$ is $1, 2, 3, 4, 5, 6, 8, 10$.

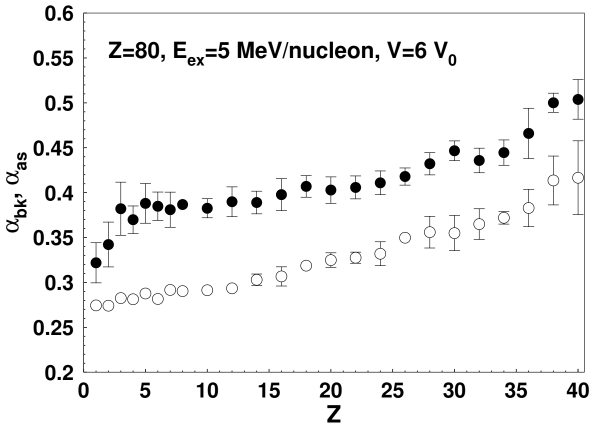

Fig. 6. Variation of the slope parameter α with Z for the fragmenting systems (185,80) and (200,80) at $V = 6V_0$ and $E_{\mathrm{ex}} = 5$ MeV/nucleon. Open symbols correspond to the primary stage, close symbols correspond to the asymptotic stage of the decay.

source's size. To check the size effect of isoscaling parameters we choose for fragmenting sources two pairs of smaller nuclei, namely ((112, 50), (124, 50)) and ((112, 50), (119, 50)) with $E_{\mathrm{ex}} = 5$ MeV/nucleon and $V = 3V_0$. The values of the slope parameters obtained in this case (((112, 50), (124, 50)): $\alpha_{bk} = 0.37$, $\alpha_{as} = 0.50$, $\beta_{bk} = -0.49$ and $\beta_{as} = -0.55$; ((112, 50), (119, 50)): $\alpha_{bk} = 0.23$, $\alpha_{as} = 0.32$, $\beta_{bk} = -0.30$ and $\beta_{as} = -0.34$) are very different from the values obtained for the $Z = 80$ sources and suggest a strong dependence of the isoscaling parameters on source size.

This result is in contradiction with ref. [11], where the canonical version of SMM [18] predicted α independent of the size of the sources. The discrepancy between the cited canonical results and the present microcanonical results focuses the attention on the importance of using appropriate models for describing finite systems.

3.8 Slope dependence on the emitted fragment size

Taking into account that isoscaling has been argued on grandcanonical basis and the above-presented microcanonical results suggest important deviations from grandcanonical expectations, it would be interesting to check whether isoscaling is valid also for smaller sources.

The results obtained for ((40, 20) and (50, 20)) at $E_{\mathrm{ex}} = 5$ MeV/nucleon and $V = 6V_0$ proved a linear behavior of the yield ratios with the isotopic composition of the emitted fragment together with a monotonic increase of the absolute value of the slope parameters with the size of the emitted fragment as displayed in fig. 5, where we plot the results obtained for fragments with $1 \leq Z \leq 10$ and $1 \leq N \leq 10$. This means that for small sources we obtain isoscaling in the sense of eqs. (7) and (8) without the constancy of the slope parameters with the size of the cluster.

This result makes us go back to large fragmenting sources in order to evidence the dependence of α and β on the size of the emitted cluster. From our analysis it results that by considering large emitted fragments isoscaling is obtained with slope parameters dependent on the size of the cluster. To illustrate this result, in fig. 6 we plot $\alpha(Z)$ for ((200, 80) and (185, 80)) at $V = 6V_0$ and $E_{\mathrm{ex}} = 5$ MeV/nucleon. One can see that while for $Z < 10$, α is constant, for larger emitted fragments α increases monotonically with Z. It is interesting to mention that isoscaling slope parameters dependent on the emitted cluster size have been evidenced also in the framework of a dynamical stochastic mean-field model [13].

Because present detectors are not able to isotopically separate fragments with charge larger than 8, the only case when one may hope to evidence this effect corresponds to light fragments emitted by light sources.

The fragment dependence of the slope parameters can be interpreted as a finite-size effect which, as expected, enhances with the diminishing of the source and with the increase of the emitted fragment.

4 Conclusions

To summarize, the present paper aims to investigate isoscaling in nuclear multifragmentation. In order to offer a complete and as accurate as possible image on the subject, the study was done using a microcanonical multifragmentation model and covered a large range of source sizes ($A = 40$–200), excitation energies ($E_{\mathrm{ex}} = 2$–15 MeV/nucleon) and freeze-out volumes ($V = 3V_0$–$10V_0$). Primary and asymptotic stages of the decay have been considered.

The results indicate that although in all considered cases the logarithm of the isotopic yields corresponding to the two different sources have a linear behavior as a function of Z and N, the slope parameters are strongly dependent on most observables characterizing the state of the sources. The only observable not affecting α and β is the freeze-out volume. This result could be of particular importance for experiments aiming to extract the asymmetry term of the binding energy using multifragmentation data, since most probably equilibrated excited systems formed in heavy ion collisions have a distribution of volumes and the event separation as a function of volume is not by far possible. The isoscaling slope parameters are affected also by sequential particle emission.

Deviations from strict isoscaling have been observed for heavy clusters originating from large multifragmenting systems and light clusters emitted by light systems and can be interpreted as finite-size effects. Such deviations from grandcanonical predictions are expected to manifest especially for small systems and stress the importance of using approximations suitable to the physical situation.

Since the microcanonical ensemble is by principle the most appropriate statistical tool to describe the disintegration of small isolated systems and microcanonical multifragmentation models in general proved to be able to describe with accuracy experimental data, the most reliable description of isoscaling is expected to be obtained with this kind of models. The accuracy of isoscaling parameters is essential for obtaining correct information on fundamental quantities like the asymmetry term of the binding energy.

References

1. H.S. Xu *et al.*, Phys. Rev. Lett. **85**, 716 (2000).
2. M.B. Tsang, W.A. Friedman, C.K. Gelbke, W.G. Lynch, G. Verde, H.S. Xu, Phys. Rev. C **64**, 041603(R) (2001).
3. J. Brzychczyk *et al.*, Phys. Rev. C **47**, 1553 (1993).
4. V. Volkov, Phys. Rep. **44**, 93 (1978).
5. G.A. Souliotis *et al.*, Phys. Rev. C **68**, 024605 (2003).
6. M. Veselsky, G.A. Souliotis, M. Jandel, Phys. Rev. C **69**, 044607 (2004).
7. M.B. Tsang, W.A. Friedman, C.K. Gelbke, W.G. Lynch, G. Verde, H.S. Xu, Phys. Rev. Lett. **86**, 5023 (2001).
8. A.S. Botvina, O.V. Lozhkin, W. Trautmann, Phys. Rev. C **65**, 044610 (2002).
9. E. Geraci *et al.*, Nucl. Phys. A **732**, 173 (2004).
10. D.V. Shetty, S.J. Yennello, A.S. Botvina, G.A. Souliotis, M. Jandel, E. Bell, A. Keksis, S. Soisson, B. Stein, J. Iglio, nucl-ex/0406008.
11. M.B. Tsang *et al.*, Phys. Rev. C **64**, 054615 (2001).
12. A. Ono, P. Danielewicz, W.A. Friedman, W.G. Lynch, M.B. Tsang, Phys. Rev. C **68**, 051601 (2003).
13. T.X. Liu *et al.*, Phys. Rev. C **69**, 014603 (2004).
14. J.P. Bondorf, A.S. Botvina, A.S. Iljinov, I.N. Mishustin, K. Sneppen, Phys. Rep. **257**, 133 (1995).
15. D.H.E. Gross, Rep. Prog. Phys. **53**, 605 (1990).
16. Al.H. Raduta, Ad.R. Raduta, Phys. Rev. C **65**, 054610 (2002).
17. S. Albergo, S. Costa, E. Costanzo, A. Rubbino, Nuovo Cimento A **89**, 1 (1985).
18. A. Majumder, S. Das Gupta, Phys. Rev. C **61**, 034603 (2000).
19. W.A. Friedman, Phys. Rev. C **42**, 667 (1990).
20. Al.H. Raduta, Ad.R. Raduta, Phys. Rev. C **55**, 1344 (1997).
21. G. Fai, J. Randrup, Nucl. Phys. A **381**, 557 (1982).
22. G. Audi, A.H. Wapstra, Nucl. Phys. A **595**, 409 (1995).
23. A.S. Iljinov *et al.*, Nucl. Phys. A **543**, 517 (1992).
24. S.R. Souza *et al.*, Phys. Rev. C **67**, 051602(R) (2003).
25. W.P. Tan *et al.*, Phys. Rev. C **68**, 034609 (2003).